

Escaping from the black hole?

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ABSTRACT: We show that if there exists a special kind of Born-Infeld type scalar field, then one can send information from inside a black hole. This information is encoded in perturbations of the field propagating in non-trivial scalar field backgrounds, which serves as a "new ether". Although the theory is Lorentz-invariant it allows, nevertheless, the superluminal propagation of perturbations with respect to the "new ether". We found the stationary solution for background, which describes the accretion of the scalar field onto a black hole. Examining the propagation of small perturbations around this solution we show the signals emitted inside the horizon can reach an observer located outside the black hole. We discuss possible physical consequences of this result.

KEYWORDS: Classical Theories of Gravity, Black Holes.

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1. Introduction

During last years the scalar fields, described by the Lagrangians with a non-standard kinetic term, attracted a considerable interest. Such structures are rather common for effective fields theories. In cosmology they were first introduced in the context of k-inflation [1] and then the k-essence models were suggested for solving the cosmic coincidence problem [2]. Tachyon matter [3], ghost condensate [4] and phantom [5] can be thought as the further developments of this idea.

In some cases Lorentz invariant theories with nonlinear kinetic terms allow the superluminal propagation of perturbations on dynamical backgrounds and this may have interesting applications in cosmology [6, 7]. We would like to point out that the issue of causality is rather nontrivial in the theories with superluminal propagation and requires further investigation. For example, the Cauchy problem is well-posed not for all initial data [10, 11, 9, 8].

One of the interesting issues is the behavior of noncanonical scalar fields in the neighborhood of black holes [12, 14, 13] and in this paper we investigate the consequences of the superluminal propagation of such fields in the black hole background. In particular, we will consider a Lorentz invariant scalar field theory with Lagrangian which allows the superluminal propagation of perturbations during accretion onto black hole. Assuming that the backreaction of the scalar field on the metric is negligible we will find first the analytic solution describing the spherically symmetric accretion of the scalar field. After that we investigate the propagation of the perturbations in this background.

2. Model

Let us consider a scalar field with the action

$$S_\phi = \int d^4x \sqrt{-g} p(X), \quad (2.1)$$

where the Lagrangian density is given by

$$p(X) = \alpha^2 \left[\sqrt{1 + \frac{2X}{\alpha^2}} - 1 \right] - \Lambda. \quad (2.2)$$

It depends only on $X \equiv \frac{1}{2} \nabla_\mu \phi \nabla^\mu \phi$, and α and Λ are free parameters of the theory. Throughout the paper ∇_μ denotes the covariant derivative and we use the natural units in which $G = \hbar = c = 1$. The kinetic part of the action is the same as in [7] and for small derivatives, that is, in the limit $2X \ll \alpha^2$, it describes the usual massless free scalar field. One can prove that the theory, described by (2.2) is ghost-free.

The equation of motion for the scalar field is

$$G^{\mu\nu} \nabla_\mu \nabla_\nu \phi = 0, \quad (2.3)$$

where the induced metric $G^{\mu\nu}$ is given by

$$G^{\mu\nu} \equiv p_{,X} g^{\mu\nu} + p_{,XX} \nabla^\mu \phi \nabla^\nu \phi, \quad (2.4)$$

and $p_{,X} \equiv \partial p / \partial X$. This equation is hyperbolic and its solutions are stable with respect to high frequency perturbations provided $(1 + 2X p_{,XX} / p_{,X}) > 0$ [6, 10, 11]. This condition is always satisfied in the model under consideration. It is well known that, if $\nabla_\nu \phi$ is timelike (that is, $X > 0$ in our convention), then the field described by (2.2) is formally equivalent to a perfect fluid with the energy density $\varepsilon(X) = 2X p_{,X}(X) - p(X)$, the pressure $p = p(X)$ and the four-velocity

$$u_\mu = \frac{\nabla_\mu \phi}{\sqrt{2X}}. \quad (2.5)$$

The effective sound speed of perturbations is given by

$$c_s^2 \equiv \frac{\partial p}{\partial \varepsilon} = 1 + \frac{2X}{\alpha^2}. \quad (2.6)$$

and for $X > 0$ it always exceeds the speed of light. For the further considerations it occurs to be convenient to express the energy density and pressure in terms of this speed of sound, namely,

$$\varepsilon = \alpha^2 (1 - c_s^{-1}) + \Lambda, \quad p = \alpha^2 (c_s - 1) - \Lambda. \quad (2.7)$$

It is easy to see that the Null Energy Condition is valid and hence the black hole area theorem [15] holds.

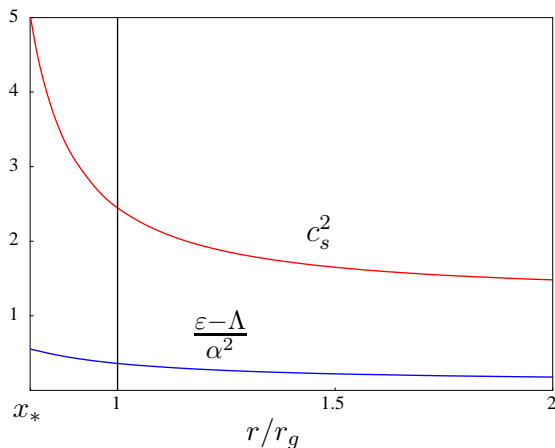


Figure 1: For the background solution in the case $c_\infty^2 = 5/4$ the squared sound speed (red) and the normalized energy density, $(\varepsilon - \Lambda)/\alpha^2$, (blue) are shown as functions of radial coordinate $x \equiv r/r_g$. The sound horizon $r_*/r_g = 4/5$ is located inside the Schwarzschild horizon.

3. Background solution

First we will find a stationary spherically symmetric background solution for the scalar field falling onto a Schwarzschild black hole. To describe the black hole we use the ingoing Eddington-Finkelstein coordinates, in which the metric takes the form:

$$ds^2 = f(r)dV^2 - 2dVdr - r^2d\Omega, \quad (3.1)$$

where $f(r) \equiv 1 - r_g/r$ and $r_g \equiv 2M$ is the gravitational radius of the black hole. The coordinate V is related to the Schwarzschild coordinates t and r as: $V \equiv t + r + r_g \ln|r/r_g - 1|$. Let us assume that the infalling field has a negligible influence on the metric, that is, we consider an accretion of the test fluid in the given gravitational field. The requirement of stationarity implies the following ansatz for the solution:

$$\phi(V, x) = \alpha \sqrt{c_\infty^2 - 1} \left(V + r_g \int F(x) dx \right), \quad (3.2)$$

where $x \equiv r/r_g$ and c_∞ is the speed of sound at infinity. The overall factor in (3.2) is chosen to recover the cosmological solution at infinity: $\phi(V, x) \rightarrow \alpha t \sqrt{c_\infty^2 - 1}$ and r_g in front of the integral is for the further convenience. The function $F(x)$ must be determined solving equations of motion for appropriate boundary conditions. Substituting (3.2) into (2.3) and integrating over r we obtain the following equation for the function $F(x)$:

$$\frac{(fF + 1)x^2}{\sqrt{1 - (fF^2 + 2F)(c_\infty^2 - 1)}} = \frac{B}{c_\infty^4}, \quad (3.3)$$

where B is the constant of integration. The solution of (3.3), which is nonsingular at the black hole horizon, is given by:

$$F(x) = \frac{1}{f} \left(B \sqrt{\frac{c_\infty^2 + f - 1}{fx^4 c_\infty^8 + B^2 (c_\infty^2 - 1)}} - 1 \right). \quad (3.4)$$

The speed of sound can then be found using (3.4), (3.2) and (

$$c_s^2 = \frac{x^3 c_\infty^8 (x c_\infty^2 - 1)}{(x - 1)x^3 c_\infty^8 + B^2 (c_\infty^2 - 1)}. \quad (3.5)$$

Note that the speed of sound becomes infinite at some $x \equiv x_{\text{sing}}$ and this singularity is physical if the real regular solution (3.4) exists for all $x > x_{\text{sing}}$.

A constant of integration B , entering (3.4) and (3.5), determines the energy flux falling onto the black hole. To fix it we have to find the solution which is non-singular on the *sound horizon* and outside it. Below we consider the propagation of perturbations and find how the position of the sound horizon depend on B . Then, given c_∞ , and comparing the positions of the singularities and the sound horizon we determine the unique value for B .

4. Small perturbations

Let us now consider the small perturbations around background (3.2), (equation (2.3) satisfy the following equation (see, e.g. [10, 11]):

$$\tilde{G}_{\mu\nu} \eta^\mu \eta^\nu = 0, \quad (4.1)$$

where $\tilde{G}_{\mu\nu}$ is the matrix inverse to $G^{\mu\nu}$, that is, $G^{\mu\nu} \tilde{G}_{\nu\sigma} = \delta_\sigma^\mu$, and it is calculated for the background solution (3.2), (3.4). The vector η^μ describes the propagation of the wave front. After lengthy, but straightforward calculations, we obtain from (4.1) and (2.4) the following differential equation for the characteristics $\eta_\pm(x) \equiv dV/dx$:

$$\eta_\pm = \frac{1}{f} + \frac{1}{\xi_\pm}, \quad (4.2)$$

where

$$\xi_\pm = \pm f \sqrt{c_\infty^2 - \frac{1}{x} \frac{\sqrt{B^2(c_\infty^2 - 1) + c_\infty^8 x^4 f}}{c_\infty^4 x^2 f \mp B(c_\infty^2 - 1)}}. \quad (4.3)$$

It is worth mentioning that the equation $\xi_\pm = dx/dt$ determines the propagation of wave front in the Schwarzschild coordinates x and t .

Equation (4.2) does not specify the *direction* of the propagation completely. In addition to the value of dV/dx one has to choose a cone of *future* and a cone of *past* for every event. However, the position of the past and the future lightcones helps us to fix the past and the future cones for the scalar field perturbations, or in other words, for the "sound". Using characteristics (4.2) we then select uniquely the sonic cones as follows: i) the past and the future sonic cones should not have overlapping regions; ii) the future sonic cone contains the future light cone, while the past sonic cone contains the past light cone. This last property can be justified because it holds at the spatial infinity and the sonic characteristics (4.2) nowhere coincide with the radial light geodesics (otherwise for the sonic signal ds^2 would vanish somewhere and this is obviously not true). As a result we conclude: a signal propagating along η_+ points in the positive V -direction, while a signal corresponding to η_- points in the negative V -direction (see figure 2).

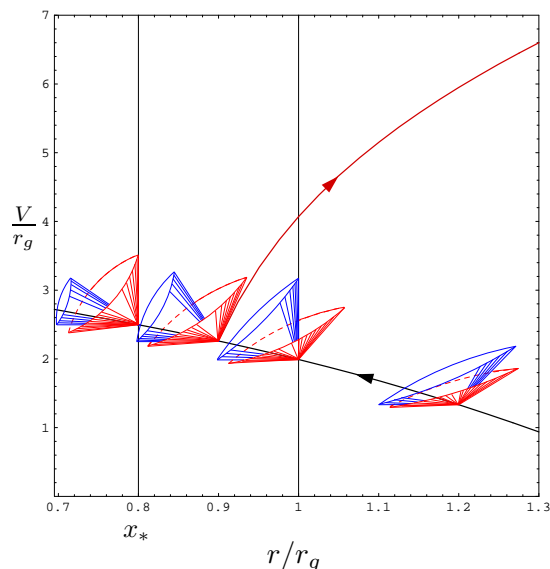


Figure 2: In the Eddington-Finkelstein coordinates the emission of a sound signal from the falling spacecraft is shown. The blue cones correspond to the future light cones and the red cones are the future sonic cones (4.2). The black curve represents the world line obtained numerically from eqs. (2.5), (3.2), (3.4) for the spacecraft which moves together with a falling background field. Being between the the Schwarzschild ($r = r_g$) and sound ($r = r_*$) horizons the spacecraft emits an acoustic signal (shown by red) which reaches the distant observer in finite time. The trajectory of the signal is obtained by the numerical integration of eq. (4.2).

Having calculated the propagation vectors we can find the *sonic horizon*. The *sonic horizon* is defined as a surface, where the length of the spatial velocity vector is equal to the speed of sound. Outside this surface the signals can reach the spatial infinity, while sound cannot escape from inside because it is trapped by the supersonic motion of a fluid (in the same way as light is trapped inside the event horizon by the gravitational field). The acoustic signal directed *out* of the black hole corresponds to η_+ and therefore the *sound horizon* is located at $x \equiv x_*$ where $\eta_+ \equiv (dV/dx)_+$ becomes infinite (see figure 2). Now we can fix a constant of integration B , entering (3.2), (3.4). We simply demand that in the physically occurring situation there exists no singularity on the *sound horizon* and outside of it. This procedure is similar to that one arising in the problem of perfect fluid accretion where the physical solution does not diverge at the event horizon (see, e.g. [16]). Thus, fixing B reduces to the analysis of the mutual location of x_{sing} and x_* . After some calculations we find the following:

- For $B \neq 1$ either the physical singularity coincides with the sound horizon or the speed of sound becomes imaginary (this means absolute instability) within some region outside the singular surface, for $x > x_{\text{sing}}$. In both cases the solution is nonphysical.
- For $B = 1$ and $c_\infty^2 > 4/3$ the speed of sound becomes imaginary before reaching of sound horizon or singularity. This solution is also nonphysical.

- For $B = 1$ and $c_\infty^2 < 4/3$ the sound horizon is located at $x_* = 1/c_\infty^2$ and the singularity is hidden inside the sound horizon, $x_{\text{sing}} < x_*$. This is the only physically relevant solution we are searching for.

Thus, we have to set $B = 1$ in (3.2), (3.4) and this ends the constructing of the background.

Before we turn to the discussion of the signals propagation we will briefly analyze the validity of the stationarity approximation when the backreaction can be neglected. Having fixed B the rate of the accretion can be easily evaluated as (see e.g. [14]):

$$\dot{M} = 4\pi M^2 \alpha^2 (c_\infty^2 - 1) / c_\infty^4. \tag{4.4}$$

It is clear that for any fixed value of c_∞ we can choose a small enough α , so that the energy flux onto black hole is negligible. The propagation of perturbations (4.2) on the background (3.2) does not depend on α , but only on c_∞ . Therefore, we can always take sufficiently small α in (2.2) to ensure that during the gedanken experiment with sending signals from the interior of a black hole the background solution remains nearly unchanged.

After we have found the physically relevant background solution we will discuss whether the acoustic signals can really escape from the interior of the black hole. This becomes possible because in the case under consideration the sound horizon ($x_* = 1/c_\infty^2$) is located inside the Schwarzschild radius. As long as the signals are emitted at large enough x , namely, at $x > x_*$, they reach the spatial infinity propagating along η_+ . For example, at the event horizon we have:

$$\eta_{\pm H} = \frac{1}{2} \frac{(c_\infty^4 \pm 1)^2}{c_\infty^2 - 1}. \tag{4.5}$$

The propagation vector η_{+H} is positive and so signals could freely penetrate the Schwarzschild horizon and move *outside* the black hole. The figure 2 shows how the acoustic signals go out from the interior of a black hole.

Let us calculate the redshift of the emitted signal. Suppose that a spacecraft moves together with the falling background field (such that in the spacecraft's system of coordinates $\nabla\phi_0 = 0$) and sends the acoustic signals with the frequency ω_0 . After a simple geometrical exercise in the plane (V, x) one can obtain that an observer at rest at the spatial infinity will detect these signals at the frequency:

$$\omega_\infty = \omega_0 \frac{c_\infty^8 x^3 (x - 1) + c_\infty^2 - 1}{x^2 c_\infty^4 (x^2 c_\infty^4 + 1)}. \tag{4.6}$$

Note that the ratio ω_∞/ω_0 is finite for any $x > x_*$ and it vanishes for $x = x_*$.

5. Conclusion

The main result of this paper can be summarized as follows: if there exist a specific Born-Infeld type fields, then during accretion of these fields onto black hole one can send information from the interior of the black hole. We would like to stress that this result has a classical origin and no quantum phenomena are involved. The discussed effect changes

the universal meaning of the Schwarzschild horizon as an event horizon and may have important consequences for the thermodynamics of black holes.

We consider the present work as simply an illustration of a concept. The particular theory examined does not have any justification from the point of view of particle physics. However, for a wide class of nonlinear theories the situation can be similar and therefore it is quite possible that the information can really be send from inside the black hole.

Also we would like to point out that in our model the cosmic censorship hypothesis is holds because the singularity is hidden by the sound horizon. The null energy condition is not violated as well. Hence the Schwarzschild horizon never decreases.

Note: the recent paper [17] deals with thermodynamics of black holes in the presence of superluminal fields. However, the model analyzed in this paper is completely different from ours, namely, the authors of [17] have considered two kinetically coupled fields, one of which is the ghost condensate [4].

Note added in proof: the similar possibility of sending signals from the inside of a black hole opens in bigravity theories [18].

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References

- [1] C. Armendariz-Picon, T. Damour and V.F. Mukhanov, *K-inflation*, *Phys. Lett.* **B 458** (1999) 209 [[hep-th/9904075](#)].
- [2] C. Armendariz-Picon, V.F. Mukhanov and P.J. Steinhardt, *A dynamical solution to the problem of a small cosmological constant and late-time cosmic acceleration*, *Phys. Rev. Lett.* **85** (2000) 4438 [[astro-ph/0004134](#)]; *Essentials of K-essence*, *Phys. Rev.* **D 63** (2001) 103510 [[astro-ph/0006373](#)].
- [3] A. Sen, *Tachyon matter*, *JHEP* **07** (2002) 065 [[hep-th/0203265](#)].
- [4] N. Arkani-Hamed, H.-C. Cheng, M.A. Luty and S. Mukohyama, *Ghost condensation and a consistent infrared modification of gravity*, *JHEP* **05** (2004) 074 [[hep-th/0312099](#)].
- [5] R.R. Caldwell, *A phantom menace?*, *Phys. Lett.* **B 545** (2002) 23 [[astro-ph/9908168](#)].
- [6] J. Garriga and V.F. Mukhanov, *Perturbations in K-inflation*, *Phys. Lett.* **B 458** (1999) 219 [[hep-th/9904176](#)].
- [7] V.F. Mukhanov and A. Vikman, *Enhancing the tensor-to-scalar ratio in simple inflation*, *JCAP* **02** (2006) 004 [[astro-ph/0512066](#)].
- [8] A. Hashimoto and N. Itzhaki, *Traveling faster than the speed of light in non-commutative geometry*, *Phys. Rev.* **D 63** (2001) 126004 [[hep-th/0012093](#)].

- [9] A. Adams, N. Arkani-Hamed, S. Dubovsky, A. Nicolis and R. Rattazzi, *Causality, analyticity and an ir obstruction to uv completion*, [hep-th/0602178](#).
- [10] C. Armendariz-Picon and E.A. Lim, *Haloes of K-essence*, *JCAP* **08** (2005) 007 [[astro-ph/0505207](#)].
- [11] A.D. Rendall, *Dynamics of K-essence*, *Class. and Quant. Grav.* **23** (2006) 1557 [[gr-qc/0511158](#)].
- [12] S. Mukohyama, *Black holes in the ghost condensate*, *Phys. Rev. D* **71** (2005) 104019 [[hep-th/0502189](#)].
- [13] K.A. Bronnikov and J.C. Fabris, *Regular phantom black holes*, *Phys. Rev. Lett.* **96** (2006) 251101 [[gr-qc/0511109](#)].
- [14] E. Babichev, V. Dokuchaev and Y. Eroshenko, *Black hole mass decreasing due to phantom energy accretion*, *Phys. Rev. Lett.* **93** (2004) 021102 [[gr-qc/0402089](#)];
E. Babichev, V. Dokuchaev and Y. Eroshenko, *The accretion of dark energy onto a black hole*, *J. Exp. Theor. Phys.* **100** (2005) 528–538 [[astro-ph/0505618](#)].
- [15] S. W. Hawking, G. F. R. Ellis, *The large scale structure of space-time*, Cambridge, 1973.
- [16] V.S. Beskin, *2d transonic hydrodynamics in general relativity*, *Les Houches Lect. Notes* **78** (2004) 85–136 [[astro-ph/0212377](#)].
- [17] S.L. Dubovsky and S.M. Sibiryakov, *Spontaneous breaking of Lorentz invariance, black holes and perpetuum mobile of the 2nd kind*, *Phys. Lett. B* **638** (2006) 509 [[hep-th/0603158](#)].
- [18] D. Blas, C. Deffayet, J. Garriga, *Global structure of bigravity solutions*, *Class. and Quant. Grav.* **23** (2006) 1697 [[hep-th/0508163](#)].